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Swing effect of spatial soliton in second order material

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Abstract. A new transverse oscillatory behavior of spatial solitons in a second order material is presented. It is based on the property of a soliton in a transverse Gaussian refractive index profile.

Key words: spatial soliton

1. Introduction

Spatial solitons are self-trapped optical beams that propagate without changing their spatial shape, since the diffraction and the nonlinear refraction balance each other in a self-focusing medium (Zakharov and Shabat 1972).

Recently the possibility of achieving large nonlinear phase shift in the parametric interaction of a fundamental frequency (FF) with its second harmonic (SH) has attracted renewed interest (De Salvo *et al.* 1992).

In the SH generation (i.e. with no seed) a nonlinear phase shift of the FF beam accompanied by small periodic conversion may be achieved by operation at large wavevector mismatches $\Delta k = 2k_1 - k_2 \neq 0$. The nonlinear phase shift, proportional to Δk, affects each field which has a non zero input. (Re *et al.* 1995; D'Aguanno *et al.*)

Therefore for a SH generation process only FF beam undergoes nonlinear phase shift which is also known as self-phase modulation process. When an amplification process occurs a "cross phase modulation" (XPM) is present: each field undergoes a phase modulation driven by the intensity of the other field, therefore periodical conversion of energy becomes strongly affected by the input intensity ratio of FF and SH fields (Fazio *et al.* 1998).

Recently an analysis of the cross phase modulation induced gain (Boardman *et al.* 1997) has been performed under vectorial analysis of the coupled FF-SH waves. These calculations analyze transverse modulation instability of the plane-wave eigenstates in quadratically nonlinear media. It has been shown that the gain curves are significantly different at low and high input power (FF power) because of the competition between birefringence and cross phase modulation induced gain.

In this paper we study the behavior of a soliton beam in a second order material in a planar waveguide , where z is the propagation direction and x is the transverse one. In the plane x-z of the waveguide there is a transverse distribution of refractive index that follows a Gaussian curve, and where the initial position of the maximum of the intensity of the soliton (with respect to x variable) is shifted with respect to the maximum of the index profile. In this situation the beam is attracted towards the center of the index profile, acquiring a certain velocity that allows it to pass this point and to continue to move forward to the other side of the index profile, decreasing its velocity. The behavior is similar to the case in which a third order material is used (Aceves *et al.* 1988; Moloney and Adachihara 1990; Garzia *et al.* 1994; Garzia *et al.* 1997) with the advantage that the case of a cascading process absorption is avoided and therefore the threshold power is lower.

2. Transverse effect of a soliton beam in a gaussian shaped refractive index profile

It is immediate to show that in a plane wave geometry the e.m. propagation of the fundamental and of the second harmonic is described by the following nonlinear couple of equations in the X-Z plane:

$$\mathrm{i}\frac{\partial A_1}{\partial Z} - \rho_1 \frac{\partial A_1}{\partial X} + \frac{1}{2k_1}\frac{\partial^2 A_1}{\partial X^2} + KA_2A_1^* \exp(\mathrm{i}\Delta kZ) = 0 \tag{1a}$$

$$\mathrm{i}\frac{\partial A_2}{\partial Z} - \rho_2 \frac{\partial A_2}{\partial X} + \frac{1}{2k_2}\frac{\partial^2 A_2}{\partial X^2} + KA_1^2 \exp(-\mathrm{i}\Delta kZ) = 0 \tag{1b}$$

where the subscripts 1 and 2 refer to the fundamental and second harmonic wave respectively, A_1, A_2 are the envelopes of the two waves, k_1, k_2 their wavevectors,

$\Delta k = 2k_1 - k_2$ is the wavevectors mismatch,

$$K = \frac{\omega_0}{c}\left(\frac{2}{c\varepsilon_0 n_f^2(\omega_0)n_s(2\omega_0)}\right)^{1/2}\frac{\chi_{\mathrm{eff}}^{(2)}}{2}$$

is the coupling constant, $n_f(w_0), n_s(2\omega_0)$ being the refractive indices at ω_0 and $2\omega_0$ respectively and ρ_1, ρ_2 the walk off angles.

We suppose the beams to propagate in a transverse refractive index gradient, that is:

$$n_f(\omega_0) = n_f^0\big(1 + \Delta n_f(x)\big) \tag{2b}$$

$$n_s(2\omega_0) = n_s^0(1 + \Delta n_s(x)), \tag{2b}$$

where $\Delta n_f(x)$, $\Delta n_s(x)$ are proper spatial functions whose shape can be equal. We also assume that a gradient for the susceptibility exist:

$$\chi_{\text{eff}}^{(2)} = \chi_0(1 + \Delta\chi(x)). \tag{3}$$

In this situation we obtain:

$$k_1 = k_1^0(1 + \Delta n_f(x) \tag{4a}$$

$$k_2 = k_2^0(1 + \Delta n_s(x)) \tag{4b}$$

and

$$\begin{aligned}\Delta k &= 2\frac{2\pi}{\lambda}\left[n_f^0(1 + \Delta n_f(x)) - n_s^0(1 + \Delta n_s(x))\right] \\ &= 2\frac{2\pi}{\lambda}\left[n_f^0 - n_s^0\frac{(1 + \Delta n_s(x))}{(1 + \Delta n_f(x))}\right](1 + \Delta n_f(x)) \\ &\cong 2\frac{2\pi}{\lambda}\left[n_f^0 - n_s^0\right](1 + \Delta n_f(x)) = \Delta k^0(1 + \Delta n_f(x)).\end{aligned} \tag{5}$$

The coupling constant can be written as:

$$K = K^0\frac{1 + \Delta\chi(x)}{\left[(1 + \Delta n_f(x))^2(1 + \Delta n_s(x))\right]^{1/2}}. \tag{6}$$

Equations (1), neglecting the walk off terms, become:

$$\begin{aligned}&\mathrm{i}\frac{\partial A_1}{\partial Z} + \frac{1}{2k_1^0(1 + \Delta n_f(x))}\frac{\partial^2 A_1}{\partial X^2} + K^0\frac{1 + \Delta\chi(x)}{\left[(1 + \Delta n_f(x))^2(1 + \Delta n_s(x))\right]^{1/2}}A_2A_1^* \\ &\quad\times\exp(\mathrm{i}\Delta k^0(1 + \Delta n_f(x))Z) = 0\end{aligned} \tag{7a}$$

$$\begin{aligned}&\mathrm{i}\frac{\partial A_2}{\partial Z} + \frac{1}{2k_2^0(1 + \Delta n_s(x))}\frac{\partial^2 A_2}{\partial X^2} + K^0\frac{1 + \Delta\chi(x)}{\left[(1 + \Delta n_f(x))^2(1 + \Delta n_s(x))\right]^{1/2}}A_1^2 \\ &\quad\times\exp(-\mathrm{i}\Delta k^0(1 + \Delta n_f(x))Z) = 0\end{aligned} \tag{7b}$$

Equations (1) and (7) can be written in normalized units:

$$\mathrm{i}\frac{\partial u_1}{\partial z}+\frac{1}{2}\frac{\partial^2 u_1}{\partial x^2}+u_2u_1^*=0 \tag{8a}$$

$$\mathrm{i}\frac{\partial u_2}{\partial z}+\frac{1}{2\sigma}\frac{\partial^2 u_2}{\partial x^2}+\delta k u_2+\frac{u_1^2}{2}=0, \tag{8b}$$

and introducing the transverse refractive index gradient we have:

$$\mathrm{i}\frac{\partial u_1}{\partial z}+\frac{1}{2k_1^0(1+\Delta n_f(x))}\frac{\partial^2 u_1}{\partial x^2}+K^0\frac{1+\Delta\chi(x)}{\left[(1+\Delta n_f(x))^2(1+\Delta n_s(x))\right]^{1/2}}u_2u_1^*=0 \tag{9a}$$

$$\mathrm{i}\frac{\partial u_2}{\partial z}+\frac{1}{2\sigma k_2^0(1+\Delta n_s(x))}\frac{\partial^2 u_2}{\partial x^2}+\delta k(1+\Delta n_f(x))u_2+K^0\frac{1+\Delta\chi(x)}{\left[(1+\Delta n_f(x))^2(1+\Delta n_s(x))\right]^{1/2}}\frac{u_1^2}{2}=0 \tag{9b}$$

where the walk off terms have been neglected and where $x=X/X_0$, with X_0 equal to the input beam waist, $z=Z/z_\mathrm{d}$, with z_d equal to the diffraction length, $\sigma=k_1/k_2$, $\delta k=(k_2-k_1)z_\mathrm{d}$, $u_1=\sqrt{2}z_\mathrm{d}KA_1$, $u_2=z_\mathrm{d}KA_2\exp(\mathrm{i}\delta kz)$.

When the transverse refractive index is absent we can define two conserved quantities. The normalized total power of the two beams:

$$M=\int_{-\infty}^{\infty}\left(|u_1|^2+2|u_2|^2\right)\mathrm{d}x, \tag{10}$$

and the center of gravity of the system:

$$\bar{x}=\frac{\int_{-\infty}^{\infty}x\left(|u_1|^2+2|u_2|^2\right)\mathrm{d}x}{M} \tag{11}$$

It is possible to demonstrate that, in the absence of a gradient of the transverse refractive index, also $v=\mathrm{d}\bar{x}/\mathrm{d}z$ is a constant quantity. This fact together with the existence of localized, stationary solutions of system (8), which have been given numerically, allows us to introduce the equivalent particle model as a tool to study the beams evolution.

We consider the transverse index variations to be of the form:

$$\Delta n_f = \Delta f H(x) \tag{12a}$$

$$\Delta n_s = \Delta s H(x) \tag{12b}$$

$$\Delta\chi = \Gamma H(x) \tag{12c}$$

Because of the transverse variations of the medium, the velocity v is no longer constant due to the presence of a force f acting on the center of gravity of the system given from (Garzia *et al.* 1997):

$$\begin{aligned} f = \frac{\mathrm{d}(M\bar{x})}{\mathrm{d}x} = -\frac{1}{2}\int_{-\infty}^{\infty}\left\{\left(u_1 u_1^* \frac{\partial H(x)}{\partial x}\Delta f + 2u_2 u_2^* \frac{\partial H(x)}{\partial x}\Delta s\right)(r_0 k_0)\right\}\mathrm{d}x \\ -\frac{1}{2}\int_{-\infty}^{\infty}\left\{\left(u_1^{*2}u_2 + u_1^2 u_2^*\right)\frac{\partial H(x)}{\partial x}\Gamma\right\}\mathrm{d}x \end{aligned} \tag{13}$$

where $k_0 = 2\pi/\lambda_0$ and r_0 is the transverse scale length.

Since the force is related to the potential by $f(x) = -\partial U/\partial x$, it is possible to show that the potential has the following expression:

$$\begin{aligned} U(\bar{x}) = &\frac{1}{2}(r_0 k_0)^2 \Delta f \int_{-\infty}^{\infty}(|u_1|^2)H(x)\,\mathrm{d}x + (r_0 k_0)^2 \Delta s \int_{-\infty}^{\infty}(|u_2|^2)H(x)\,\mathrm{d}x \\ &+ \Gamma\int_{-\infty}^{\infty}(u_1^{*2}u_2)H(x)\,\mathrm{d}x + \Gamma\int_{-\infty}^{\infty}(u_1^2 u_2^*)H(x)\,\mathrm{d}x, \end{aligned} \tag{14}$$

that is the potential depends of both the amplitudes of the fundamental and of the second harmonic and of the profile $H(x)$.

If the initial profiles of the input beams are:

$$u_1(x,0) = u_1^0 \exp\left[-\left(\frac{x-\bar{x}}{2\alpha}\right)^2\right] \tag{15a}$$

$$u_2(x,0) = u_2^0 \exp\left[-\left(\frac{x-\bar{x}}{2\beta}\right)^2\right] \tag{15b}$$

where α and β are constants, Equation (14) is composed from a series of terms as:

$$U_i(\bar{x}) = k_i \int_{-\infty}^{\infty} \exp\left[-a_i(x-\bar{x})^2\right] H(x)\,\mathrm{d}x \tag{16}$$

where

$$k_1 = \frac{1}{2}(u_1^0)^2 \Delta f (r_0 k_0)^2, \quad k_2 = (u_2^0)^2 \Delta s (r_0 k_0)^2, \quad k_3 = 2\Gamma (u_1^0)^2 u_2^0,$$
$$a_1 = \frac{1}{2\alpha^2}, \quad a_2 = \frac{1}{2\beta^2}, \quad a_3 = \frac{1}{2\alpha^2} + \frac{1}{4\beta^2}.$$

If $H(x) = \exp(-bx^2)$, that is a Gaussian refractive index profile, where b is a parameter responsible for the width of the profile, the terms of the potential can be written as:

$$\begin{aligned} U_i(\bar{x}) &= k_i \int_{-\infty}^{\infty} \exp\left[-a_i (x - \bar{x})^2\right] \exp(-bx^2)\, \mathrm{d}x \\ &= k_i \mathrm{F}^{-1}\left[\sqrt{\frac{\pi}{a_i}} \exp\left(-\frac{\omega^2}{4a_i}\right) \sqrt{\frac{\pi}{b}} \exp\left(-\frac{\omega^2}{4b}\right)\right] \\ &= k_i \frac{\pi}{\sqrt{a_i b}} \mathrm{F}^{-1}\left[\exp\left(-\frac{\omega^2}{4}\left(\frac{1}{a_i} + \frac{1}{b}\right)\right)\right] \\ &= k_i \frac{\pi}{\sqrt{a_i b}} \mathrm{F}^{-1}\left[\exp\left(-\frac{\omega^2}{4a_i'}\right)\right] = k_i \sqrt{\frac{\pi}{a_i + b}} \exp\left(-a_i' \bar{x}^2\right) \end{aligned} \tag{17}$$

where $a_i' = (a_i b)/(a_i + b)$ and $\mathrm{F}^{-1}[\bullet]$ is the inverse Fourier transform.

The acceleration that acts on the beams can thus be rapidly calculated as:

$$a(\bar{x}) = \frac{f(\bar{x})}{M} = -\frac{1}{M}\frac{\partial U(\bar{x})}{\partial \bar{x}} = \frac{2}{M}\sum_i k_i \sqrt{\frac{\pi}{a_i + b}} a_i' \bar{x} \; \exp(-a_i' \bar{x}^2), \tag{18}$$

where

$$M = \int_{-\infty}^{+\infty} |u_1(x,z)|^2 + 2|u_2(x,z)|^2 = \sqrt{2\pi}\left[\alpha (u_1^0)^2 + 2\beta (u_2^0)^2\right]. \tag{19}$$

If $b \ll a_i$ Equation (18) can be approximated as:

$$a(\bar{x}) = \frac{2}{M}\sum_i k_i \sqrt{\frac{\pi}{a_i}} b\bar{x} \; \exp(-b\bar{x}^2). \tag{20}$$

The acceleration expressed from Equation (20) has a form that is equal to the acceleration found in the third order medium (Garzia *et al.* 1997) even if it is quite complex due also to the greater number of parameters involved in this process.

If the beams are positioned in the point where the acceleration is maximum, that is $x_M = 1/\sqrt{2b}$, the resulting oscillation period is:

$$T = \left(\frac{32}{\frac{2}{M} \sum_i k_i b \sqrt{\frac{\pi}{a_i}} (1 - \exp(-1/2))} \right)^{1/2}. \tag{21}$$

The above theory has been confirmed through numerical simulations, using a BPM algorithm, of Equation (9) and the results are shown in Fig. 1. The plot

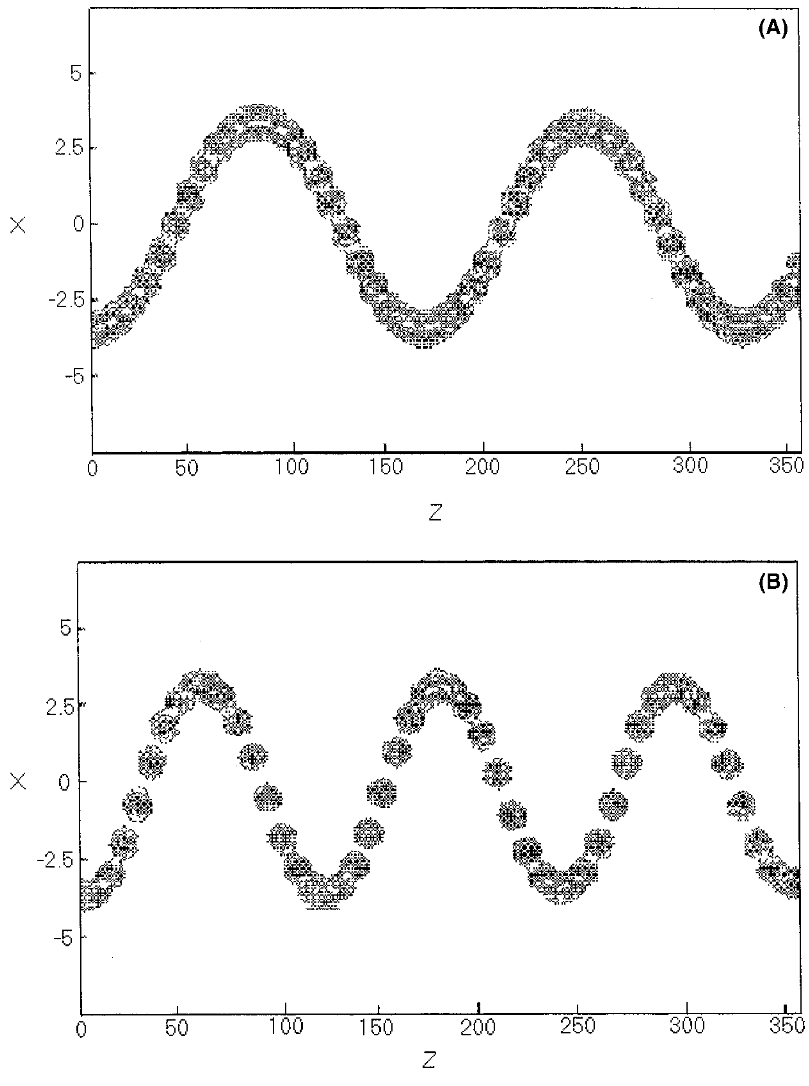

Fig. 1. (A) Upper view of a numerical simulation of the fundamental beam for $\Gamma = 0.05$, $\Delta f = \Delta s = 0.05$, $u_1^0 = 4.2$, $u_2^0 = 3$, $\alpha = 0.25$, $\beta = 0.25$, $b = 0.056$, $\lambda_0 = 1.18 \times 10^{-6}$ m, $r_0 = 1 \times 10^{-7}$. (B) Upper view of a numerical simulation of the fundamental beam for $\Gamma = 0.05$, $\Delta f = \Delta s = 0.05$, $u_1^0 = 5.6$, $u_2^0 = 4$, $\alpha = 0.25$, $\beta = 0.25$, $b = 0.056$, $\lambda_0 = 1.18 \times 10^{-6}$ m, $r_0 = 1 \times 10^{-7}$.

has been compressed through the use of a double scale in the axes, to let the effect be more evident. The inclination angle is therefore not as large as the geometrical angle of the plot, letting us use an ordinary BPM algorithm that would otherwise be unusable. The discontinuity of the plot is due to the conversion of the energy from the fundamental to the second harmonic and vice versa. It is possible to see that the beams oscillate according to the period analytically expressed from Equation (21).

The situation is quite similar to the third order material case. The main difference is that in this case we observe both the oscillation of the fundamental and the second order beams during their reciprocal conversion.

3. Conclusions

The studied behavior of a localized beam in a Gaussian shaped waveguide allows the swing effect to take place even in a second order material. The oscillation period depends on both the amplitude of the beams and on the parameters of the waveguide as it happens in third order material case.

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